Universidade Federal de Pernambuco
Projeto Estrutural
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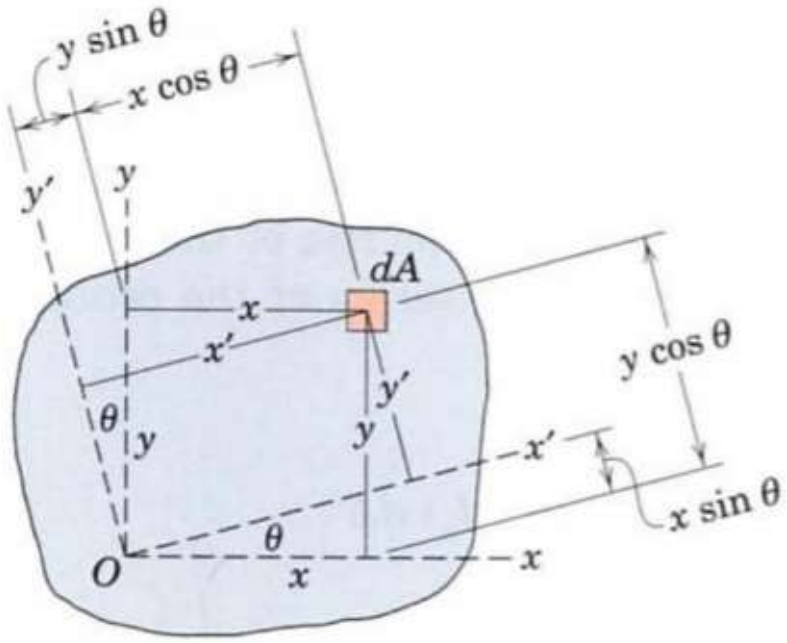
Rotação de inércia




Leis de transformaçao

Matriz de rotação: $\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$

$$x' := x \cdot \cos(\theta) + y \cdot \sin(\theta)$$
$$y' := y \cdot \cos(\theta) - x \cdot \sin(\theta)$$

$$Ix' := \int y'^{2}\, dA \qquad Iy' := \int x'^{2}\, dA \qquad Ix'y' := \int (x \cdot y)\, dA$$

$$dIx' := y'^{2} \cdot dA \qquad dIy' := x'^{2} \cdot dA \qquad dIx'y' := x' \cdot y' \cdot dA$$

$$dIx' := (y \cdot \cos(\theta) - x \cdot \sin(\theta))^{2} \cdot dA$$

$$dIx' := (y \cdot \cos(\theta) + x \cdot \sin(\theta))^{2} \cdot dA$$

$$dIx'y' := (y \cdot \cos(\theta) - x \cdot \sin(\theta)) \cdot (y \cdot \cos(\theta) + x \cdot \sin(\theta)) \cdot dA$$

$$dIx' := \left( (y \cdot \cos(\theta))^2 - 2 \cdot y \cdot \cos(\theta) \cdot x \cdot \sin(\theta) + (x \cdot \sin(\theta))^2 \right) \cdot dA$$

$$dIy' := \left( (y \cdot \cos(\theta))^2 + 2 \cdot y \cdot \cos(\theta) \cdot x \cdot \sin(\theta) + (x \cdot \sin(\theta))^2 \right) \cdot dA$$

$$dIx'y' := (y \cdot \cos(\theta))^2 + (x \cdot \sin(\theta))^2$$

Sabendo que:

$$\cos(x)^2 := \frac{1}{2} \cdot (1 + \cos(2 \cdot x))$$

$$\sin(x)^2 := \frac{1}{2} \cdot (1 - \cos(2 \cdot x))$$

$$\sin(2 \cdot x) := 2 \cdot \sin(x) \cdot \cos(x)$$

Substituindo e integrando novamente, teremos:

$$Ix' := Ix \cdot \cos(\theta)^2 + Iy \cdot \sin(\theta)^2 - 2 \cdot Ixy \cdot \sin(\theta) \cdot \cos(\theta)$$

$$Iy' := Ix \cdot \sin(\theta)^2 + Iy \cdot \cos(\theta)^2 + 2 \cdot Ixy \cdot \sin(\theta) \cdot \cos(\theta)$$

$$Ix'y' := Ix \cdot \sin(\theta) \cdot \cos(\theta) - Iy \cdot \sin(\theta) \cdot \cos(\theta) + Ixy \cdot \left( \cos(\theta)^2 - \sin(\theta)^2 \right)$$